On the difference between proton and neutron spin-orbit splittings in nuclei

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Abstract. The latest experimental data on nuclei at ^{132}Sn permit us for the first time to determine the spin-orbit splittings of neutrons and protons in identical orbits in this neutron-rich doubly magic region and compare the case to that of ^{208}Pb. Using the new results, which are now consistent for the two neutron-rich doubly magic regions, a theoretical analysis defines the isotopic dependence of the mean-field spin-orbit potential and leads to a simple explicit expression for the difference between the spin-orbit splittings of neutrons and protons. The isotopic dependence is explained in the framework of different theoretical approaches.

1 Introduction

Spin-orbit splitting of the mean-field orbitals is one of the main factors, which determine the nuclear structure in nuclei both near and far from the closed shells. While the global characteristics of the spin-orbit splitting are well known, one cannot say the same about the isotopical dependence of the splitting. However, the new experimental results obtained recently [1–3] on nuclei close to ^{132}Sn allowed to define a nearly complete set of neutron and proton single-particle orbitals and some important statical and dynamical properties of the mentioned nuclides. In particular, from the measurements on ^{133}Sb new information was obtained [2] on the energies and the decay properties of proton single-particle states above the $Z = 50, N = 82$ shells. One purpose of our study is to evaluate the new results on the single-particle levels at ^{132}Sn and intercompare them to the known data on such states in the other doubly closed-shell (DCS) regions. However, our main aim is to examine the magnitude of the spin-orbit splittings of neutrons and protons in identical orbits, and to determine their isospin dependence. A preliminary account of this work was given in [4].

The determination of the isospin dependence has a broader significance, since the magnitude of the spin-orbit splittings could be one of the factors contributing to significant structural changes in nuclides having an extreme neutron excess. Consequently, the trends extracted from the empirical data presently available are crucial guidelines for the theoretical analysis, and it is important to show that model calculations indeed do reproduce such trends.

The following presentation begins with an analysis of the existing experimental data in sect. 2, followed by theoretical considerations and evaluations in sect. 3. A summary and conclusions are included in the last section.

2 Experimental values

We now examine the experimental data on the spin-orbit splitting for neutrons and protons for two groups of the doubly closed-shell nuclei: the $N = Z$ and the neutron-rich regions. In order to facilitate the comparison, the available systematics of single-particle energies at the $N = Z$ nuclei of ^{16}O, ^{40}Ca, ^{100}Sn, and the neutron-rich ^{132}Sn and ^{208}Pb are presented in tables 1 to 5, respectively. Here, the energies of the particle and hole states closest to the Fermi level were determined from the differences of binding energies of the core and the corresponding adjacent odd nuclei: $\varepsilon(\text{particle}) = B(\text{core}) - B(\text{core} + \text{nucleon})$ and $\varepsilon(\text{hole}) = B(\text{core} - \text{nucleon}) - B(\text{core})$, using the experimental binding energies from [5]. The energies of orbitals more distant from the Fermi level were subsequently defined by the addition (subtraction) of the experimentally

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determined excitation energies [1,2,6–11] of the corresponding orbitals in the adjacent odd nuclei. Important for this process is that we have accounted for the fragmentation of states in the cases when the pertinent data were available. The cases where the effect is essential are indicated in tables 1–5 by an asterisk (*) next to the experimental value.

2.1 N = Z doubly closed-shell regions

The spin-orbit splittings in the $N = Z$ nuclei can be determined using the data in tables 1–3. Here the spin-orbit splittings of the $1p_{1/2}$-$1p_{3/2}$ and $1d_{3/2}$-$1d_{5/2}$ orbits in ^{16}O [12,13] are practically equal for protons and neutrons, with the values of 6.32 and 6.17 MeV (the difference being -2.4%) and 5.00 and 5.08 MeV (the difference being $+1.6\%$). Similarly, for ^{40}Ca, ref. [14], we have the values for protons and neutrons equal to 2.01 and 2.00 MeV for

Table 1. Single-particle levels of ^{16}O. Set 1: $V_0 = -52.21$ MeV, $V_{\ell s} = 28.6$ MeV · fm^2, $a_p = 0.67$ fm, $a_n = 0.55$ fm are fixed. Set 3: $V_0 = -51.40$ MeV, $V_{\ell s} = 25.7$ MeV · fm^2, $a_p = 0.45$ fm, $a_n = 0.50$ fm.

$n\ell j$	$\varepsilon_{\exp}$	Stnd	Set 1	Set 3	SIII-1	SIII-2
$\nu 1d_{3/2}$	(0.94)	0.89	0.18	0.20	0.66	0.67
$\nu 2s_{1/2}$	−3.27	−3.59	−3.89	−3.31	−2.88	−2.87
$\nu 1d_{5/2}$	−4.14	−6.97	−6.85	−6.41	−6.87	−6.89
$\nu 1p_{1/2}$	−15.67	−15.06	−16.05	−16.33	−14.58	−14.56
$\nu 1p_{3/2}$	(−21.84)	−19.98	−20.25	−20.10	−20.58	−20.59
$\pi 1d_{3/2}$	(4.40)	3.76	2.92	3.48	3.55	3.56
$\pi 2s_{1/2}$	−0.11	−0.89	−1.14	0.22	0.03	0.03
$\pi 1d_{5/2}$	4−0.60	−2.76	−2.67	−2.97	−3.57	−3.59
$\pi 1p_{1/2}$	−12.13	−9.95	−10.87	−12.60	−11.17	−11.15
$\pi 1p_{3/2}$	(−18.45)	−14.66	−14.90	−16.40	−17.07	−17.08

Table 2. Single-particle states of ^{40}Ca. Set 1: $V_0 = -52.39$ MeV, $V_{\ell s} = 27.9$ MeV · fm^2; $a_p = 0.67$ fm and $a_n = 0.55$ fm are fixed. Set 3: $V_0 = -52.95$ MeV, $V_{\ell s} = 28.2$ MeV · fm^2, $a_p = 0.63$ fm, $a_n = 0.68$ fm. Experimental single-particle energy marked by an asterisk (*) represents a mean value weighted by the spectroscopic factors.

$n\ell j$	$\varepsilon_{\exp}$	Stnd	Set 1	Set 3	SIII-1	SIII-2
$\nu 1f_{5/2}$	−3.48	−2.57	−3.91	−3.54	−1.49	−1.48
$\nu 2p_{1/2}$	−4.42	−3.35	−4.08	−4.69	−2.20	−2.23
$\nu 2p_{3/2}$	−6.42	−5.71	−6.08	−6.57	−4.09	−4.05
$\nu 1f_{7/2}$	−8.36	−10.43	−10.44	−9.72	−9.92	−9.94
$\nu 1d_{3/2}$	−15.64	−16.21	−17.40	−16.43	−15.53	−15.54
$\nu 2s_{1/2}$	−18.11	−16.51	−17.17	−17.00	−15.94	−15.92
$\nu 1d_{5/2}$	−21.64*	−21.08	−21.44	−20.52	−21.90	−21.90
$\pi 1f_{5/2}$	3.86	4.92	3.79	3.41	4.90	4.91
$\pi 2p_{1/2}$	2.64	2.62	2.11	2.07	3.66	3.64
$\pi 2p_{3/2}$	0.63	0.89	0.60	0.45	2.23	2.26
$\pi 1f_{7/2}$	−1.09	−2.19	−2.18	−2.85	−3.04	−3.06
$\pi 1d_{3/2}$	−8.33	−7.11	−8.25	−9.01	−8.52	−8.53
$\pi 2s_{1/2}$	−10.85	−8.18	−8.78	−9.30	−8.77	−8.75
$\pi 1d_{5/2}$	−14.33*	−12.05	−12.36	−13.19	−14.74	−14.75

Table 3. Single-particle states of ^{100}Sn. Set 1: $V_0 = -51.97$ MeV, $V_{\ell s} = 33.5$ MeV·fm^2; $a_p = 0.67$ fm and $a_n = 0.55$ fm are fixed. Set 3: $V_0 = -51.40$ MeV, $V_{\ell s} = 35.6$ MeV · fm^2, $a_p = 0.52$ fm, $a_n = 0.56$ fm.

$n\ell j$	ε_{sys}	Stnd	Set 1	Set 3	SIII-1	SIII-2
$\nu 1h_{11/2}$	−8.6(5)	−8.66	−9.01	−8.72	−6.35	−6.87
$\nu 2d_{3/2}$	−9.2(5)	−8.90	−9.24	−8.70	−7.84	−7.66
$\nu 3s_{1/2}$	−9.3(5)	−9.16	−9.53	−9.13	−7.58	−7.52
$\nu 1g_{7/2}$	−10.93(20)	−11.64	−12.02	−11.23	−10.33	−9.63
$\nu 2d_{5/2}$	−11.13(20)	−11.62	−11.97	−11.59	−10.07	−10.10
$\nu 1g_{9/2}$	−17.93(20)	−17.23	−17.61	−17.21	−16.54	−17.00
$\nu 2p_{1/2}$	−18.38(20)	−19.14	−19.53	−18.93	−19.08	−18.93
$\pi 1g_{7/2}$	3.90(15)	3.88	3.54	2.70	3.38	4.04
$\pi 2d_{5/2}$	3.00(80)	2.74	2.45	2.64	3.70	3.69
$\pi 1g_{9/2}$	−2.92(20)	−2.01	−2.36	−3.66	−2.74	−3.16
$\pi 2p_{1/2}$	−3.53(20)	−3.48	−3.84	−3.94	−4.80	−4.65
$\pi 2p_{3/2}$	−6.38	−4.95	−5.31	−5.55	−6.22	−6.18
$\pi 1f_{5/2}$	−8.71	−5.54	−5.92	−7.60	−8.43	−7.89

the $2p_{1/2}$-$2p_{3/2}$ orbit, 6.00 and 6.00 MeV for the $1d_{3/2}$-$1d_{5/2}$ orbit, and 4.95 and 4.88 MeV for the $1f_{5/2}$-$1f_{7/2}$ case. In the absence of experimental data on the single-particle states at ^{100}Sn, we adopt here the extrapolated single-particle energies from Grawe *et al.* [15], as is shown in table 3. Based on these data, one may conclude that, within the errors, the spin-orbit splitting of the $1g_{7/2}$-$1g_{9/2}$ orbit is also equal for protons and neutrons, namely, at 6.82(28) and 7.00(28) MeV. These six cases show that the splitting for the $N = Z$ DCS regions is practically equal, with small oscillations either way, but below 2.6%. This equality simply reflects the concept of isobaric invariance in nuclei.

2.2 Neutron-rich doubly closed-shell regions

For the neutron-rich nuclei we find the situation considerably different, see tables 4-5 for the data, and table 6 regarding the splittings. In the ^{132}Sn region, the energy of the $3/2^+$ proton state in ^{133}Sb was recently determined [2] at 2.44 MeV. Using this value and the previously determined single-particle excitations in nuclei close to ^{132}Sn (see [1,6–9]) the spin-orbit splittings of the $2d$ levels both in proton and neutron systems at ^{132}Sn can be now defined. The $2d_{3/2}$-$2d_{5/2}$ splitting was found to be 1.48 MeV for protons and 1.65 MeV for neutrons. This means that the neutron spin-orbit splitting is somewhat larger (by more than 11%) than for protons.

In the case of ^{208}Pb, it was noted [2] that the situation seemed to be quite opposite. Namely, a simple analysis of the single-particle levels in ^{209}Bi and ^{207}Pb suggested [2] that the spin-orbit splitting of the $2f_{5/2}$-$2f_{7/2}$ orbit is equal to 1.93 MeV for protons and 1.77 MeV for neutrons. However, a significant correction is needed. It follows from the experimental evidence that the neutron $2f_{7/2}$ state in ^{207}Pb is strongly fragmented, while the conclusions in ref. [2] were derived by considering only the lowest, albeit the strongest component of this state. In order to identify,

Table 4. Single-particle states of ^{132}Sn. Stnd: $\delta = 0.589$ MeV. Set 1: $V_0 = -51.56$ MeV, $V_{\ell s} = 33.3$ MeV $\cdot$ fm^2, $\beta = 1.39$, $\delta = 0.638$ MeV. Set 2: $V_0 = -51.44$ MeV, $V_{\ell s} = 34.8$ MeV $\cdot$ fm^2, $\beta = 1.39$, $\beta_{\ell s} = 1.35$, $\delta = 0.575$ MeV. Set 3: $V_0 = -51.55$ MeV, $V_{\ell s} = 32.4$ MeV $\cdot$ fm^2, $\beta = 1.31$, $a_p = 0.63$ fm, $a_n = 0.66$ fm, $\delta = 0.546$ MeV. Set 4: $V_0 = -51.56$ MeV, $V_{\ell s} = 34.1$ MeV $\cdot$ fm^2, $\beta = 1.34$, $\beta_{\ell s} = 1.33$, $a_p = 0.65$ fm, $a_n = 0.66$ fm, $\delta = 0.478$ MeV. Note that some theoretical works [16] postulate that the neutron $1i_{13/2}$ state at ^{132}Sn is only 1.9 MeV above the $\nu 2f_{7/2}$ level. Our calculations unequivocally demonstrate, that this state lies considerably higher, with its energy equal to $+0.55$, $+1.59$ and $+1.02$ MeV for the Stnd, SIII-1 and SIII-2 parameter sets, respectively.

$n\ell j$	$\varepsilon_{\exp}$	Stnd	Set 1	Set 2	Set 3	Set 4	SIII-1	SIII-2
$\nu 2f_{5/2}$	-0.58	0.36	0.73	0.46	0.22	-0.01	0.67	0.79
$\nu 3p_{1/2}$	(-0.92)	-0.13	-0.48	-0.09	-0.55	-0.61	0.16	0.20
$\nu 1h_{9/2}$	-1.02	-1.61	-0.84	-1.38	-0.47	-0.97	-0.72	-0.02
$\nu 3p_{3/2}$	-1.73	-0.78	-0.88	-0.77	-1.42	-1.32	-0.16	-0.14
$\nu 2f_{7/2}$	-2.58	-2.18	-2.55	-2.21	-2.84	-2.52	-1.67	-1.71
$\nu 2d_{3/2}$	-7.31	-7.74	-7.45	-7.62	-7.63	-7.77	-8.42	-8.26
$\nu 1h_{11/2}$	-7.55	-7.11	-7.96	-7.23	-7.33	-6.60	-7.69	-8.23
$\nu 3s_{1/2}$	-7.64	-7.68	-7.73	-7.64	-8.03	-7.93	-8.26	-8.21
$\nu 2d_{5/2}$	-8.96	-9.66	-9.94	-9.66	-9.98	-9.69	-10.71	-10.71
$\nu 1g_{7/2}$	-9.74	-10.56	-10.04	-10.39	-9.51	-9.81	-11.92	-11.32
$\pi 3s_{1/2}$	(-6.83)	-6.84	-6.87	-6.80	-6.64	-6.70	-4.97	-4.90
$\pi 1h_{11/2}$	-6.84	-7.32	-6.66	-7.46	-6.77	-7.48	-5.64	-6.01
$\pi 2d_{3/2}$	-7.19	-6.86	-7.20	-6.74	-7.07	-6.72	-5.93	-5.77
$\pi 2d_{5/2}$	-8.67	-9.36	-9.20	-9.37	-9.04	-9.30	-7.88	-7.88
$\pi 1g_{7/2}$	-9.63	-9.84	-10.41	-9.66	-10.60	-9.81	-10.08	-9.56
$\pi 1g_{9/2}$	-15.71	-14.91	-14.46	-15.00	-14.57	-15.02	-15.03	-15.36
$\pi 2p_{1/2}$	-16.07	-16.01	-16.22	-15.92	-16.14	-15.91	-16.68	-16.55

Table 5. Single-particle states of ^{208}Pb. The standard (Stdn) set of parameters corresponds to $V_0 = -51.50$ MeV, $V_{\ell s} = 33.2$ MeV$\cdot$fm^2, $\beta = \beta_{\ell s} = +1.39$, $a_p = 0.67$ fm, $a_n = 0.55$ fm and $\delta = 0.604$ MeV. Set 1 corresponds to $V_0 = -51.39$ MeV, $V_{\ell s} = 33.1$ MeV$\cdot$fm^2, $\beta = 1.43$ with $\beta_{\ell s} = -0.6$, $a_p = 0.67$ fm, $a_n = 0.55$ fm fixed; $\delta = 0.654$ MeV. Set 2 corresponds to $V_0 = -51.34$ MeV, $V_{\ell s} = 33.1$ MeV $\cdot$ fm^2, $\beta = 1.40$, $\beta_{\ell s} = 1.26$ with $a_p = 0.67$ fm, $a_n = 0.55$ fm fixed; $\delta = 0.593$ MeV. Set 3 corresponds to $V_0 = -51.99$ MeV, $V_{\ell s} = 32.7$ MeV $\cdot$ fm^2, $\beta = 1.36$, $a_p = 0.73$ fm, $a_n = 0.72$ fm with $\delta = 0.369$ MeV; $\beta_{\ell s} = -0.6$ is fixed. Set 4 corresponds to $V_0 = -51.93$ MeV, $V_{\ell s} = 35.2$ MeV $\cdot$ fm^2, $\beta = 1.38$, $\beta_{\ell s} = 1.76$, $a_p = 0.73$ fm, $a_n = 0.72$ fm; $\delta = 0.366$ MeV. Experimental single-particle energy marked by an asterisk (*) represents a mean value weighted by the spectroscopic factors.

$n\ell j$	$\varepsilon_{\exp}$	Stnd	Set 1	Set 2	Set 3	Set 4	SIII-1	SIII-2
$\nu 3d_{3/2}$	-1.40	-0.32	-0.02	-0.23	-0.96	-0.99	0.38	0.42
$\nu 2g_{7/2}$	-1.44	-0.79	-0.18	-0.65	-0.89	-1.14	0.01	0.14
$\nu 4s_{1/2}$	-1.90	-0.80	-0.70	-0.74	-1.63	-1.51	-0.08	-0.06
$\nu 1j_{15/2}$	-2.09^*	-2.42	-3.05	-2.31	-2.23	-1.55	-1.41	-1.93
$\nu 3d_{5/2}$	-2.37	-1.50	-1.45	-1.40	-2.35	-2.13	-0.39	-0.38
$\nu 1i_{11/2}$	-3.16	-4.24	-3.37	-4.05	-2.71	-3.33	-3.37	-2.77
$\nu 2g_{9/2}$	-3.94	-3.71	-3.82	-3.59	-4.24	-3.88	-2.91	-2.97
$\nu 3p_{1/2}$	-7.37	-7.32	-6.94	-7.17	-7.59	-7.61	-7.21	-7.13
$\nu 2f_{5/2}$	-7.94	-8.42	-7.87	-8.25	-8.17	-8.38	-8.59	-8.44
$\nu 3p_{3/2}$	-8.27	-8.18	-8.03	-8.04	-8.59	-8.43	-8.18	-8.15
$\nu 1i_{13/2}$	-9.00	-9.21	-9.62	-9.08	-8.84	-8.31	-9.73	-10.21
$\nu 2f_{7/2}$	-10.07^*	-10.57	-10.57	-10.43	-10.72	-10.46	-11.21	-11.24
$\nu 1h_{9/2}$	-10.78	-12.06	-11.35	-11.87	-10.60	-11.09	-13.16	-12.67
$\pi 3p_{1/2}$	0.17^*	0.63	0.43	0.72	0.29	0.47	2.79	2.88
$\pi 3p_{3/2}$	-0.68	-0.45	-0.46	-0.35	-0.58	-0.69	1.99	2.03
$\pi 2f_{5/2}$	-0.97	-0.68	-1.03	-0.60	-1.03	-0.61	0.60	0.74
$\pi 1i_{13/2}$	-2.19	-2.86	-2.37	-2.71	-1.94	-2.78	-1.20	-1.53
$\pi 2f_{7/2}$	-2.90	-3.38	-3.24	-3.26	-3.21	-3.53	-1.64	-1.66
$\pi 1h_{9/2}$	-3.80	-4.60	-5.11	-4.53	-4.71	-4.01	-4.68	-4.24
$\pi 3s_{1/2}$	-8.01	-7.76	-7.86	-7.67	-7.87	-7.87	-7.39	-7.33
$\pi 2d_{3/2}$	-8.36	-8.41	-8.66	-8.32	-8.59	-8.30	-8.64	-8.51
$\pi 1h_{11/2}$	-9.36	-9.33	-8.99	-9.18	-8.60	-9.21	-9.35	-9.65
$\pi 2d_{5/2}$	-10.04^*	-10.10	-10.05	-9.98	-9.96	-10.15	-10.29	-10.28
$\pi 1g_{7/2}$	-12.18^*	-12.07	-12.45	-11.99	-12.08	-11.58	-13.94	-13.59

Table 6. Magnitudes in MeV of neutron and proton spin-orbit splittings. Notation is as in previous tables. Data are given only in the cases where spin-orbit partners of both neutrons and protons in identical orbits have been observed experimentally. Note that the splittings are practically identical for neutrons and protons in the $N = Z$ nuclei, which are not included in this table.

Nucleus	$n\ell j$	$\Delta_{\exp}$	Stnd	Set 1	Set 3	SIII-1	SIII-2
^{132}Sn	$\nu 2d$	1.65	1.92	2.49	2.35	2.29	2.45
	$\pi 2d$	1.48	2.50	2.00	1.97	1.95	2.11
^{208}Pb	$\nu 2f$	2.13	2.15	2.70	2.55	2.62	2.80
	$\pi 2f$	1.93	2.70	2.21	2.18	2.24	2.40
	$\nu 3p$	0.90	0.86	1.09	1.00	0.97	1.02
	$\pi 3p$	0.85	1.08	0.89	0.87	0.80	0.85

in the spirit of refs. [17,18], the true single-particle energy of the neutron $2f_{7/2}$ state, we use the weighted average of the fragmented $7/2^-$ energy levels, with the weight provided by the spectroscopic factors determined in the (d, t) reaction on ^{208}Pb [19]. In this way we obtain a more accurate unfragmented excitation energy of this state equal to 2.70 MeV (instead of 2.34 MeV). Using this excitation energy, included in table 5, we find the neutron spin-orbit splitting of the $2f$ orbit as 2.13 MeV, which, similarly to the case of the $2d$ orbit in ^{132}Sn, is larger by about 10% than the splitting of 1.93 MeV for protons.

An additional piece of evidence along the same line is given by the analysis of the $3p_{1/2}$-$3p_{3/2}$ spin-orbit splitting at ^{208}Pb. One obtains 0.85 MeV for protons (after correcting for the fragmentation of the proton $3p_{1/2}$ level) and 0.90 MeV for neutrons. Thus again the value for neutrons is larger by about 6% than for protons. Consequently, based on the three cases described above, it is evident that the neutron spin-orbit splitting in the neutron-rich DCS nuclei of ^{208}Pb and ^{132}Sn, is *systematically larger by* $\sim 10\%$ than the corresponding proton splitting.

2.3 Fragmentation of strength at ^{132}Sn

The fragmentation of single-particle states at ^{208}Pb is mainly influenced by the presence of a very low-lying and highly collective 3_1^- phonon state at 2.62 MeV. The effects caused by the 2_1^+ state are less as the collectivization of quadrupole phonon, and the corresponding nucleon-phonon vertices are small in heavy nuclei near doubly closed shells. For example, in the neutron "hole" ^{207}Pb nuclei, the main part of energy shift of the $7/2^-$ level is caused by mixing with the higher-lying $(3_1^- \otimes \nu 1 i_{13/2}{}^{-1})_{7/2^-}$ state. Numerical evaluation performed by using the quasiparticle-phonon model with the coupling constant extracted from the $B(E3; 3_1^- \rightarrow$ ground state) value shows that the $7/2^-$ level corresponding to the "pure" $\nu 2f_{7/2}{}^{-1}$ state moves down by the amount of ~ 0.4 MeV, thus approaching the experimental value of 2.34 MeV. This large shift is due to rather strong coupling

constant and non-spin-flip nature of the matrix element. The magnitude of the predicted shift is very close to the experimental value of $2.70 - 2.34 = 0.36$ MeV mentioned above. At the same time, the $(3_1^- \otimes \nu 1 i_{13/2}{}^{-1})$ configuration has no $5/2^-$ component and thus one does not observe experimentally the fragmentation of the lower-lying $5/2^-$ level at 0.57 MeV. We note here that the $7/2^-$ and the $5/2^-$ states in the proton "particle" ^{209}Bi nucleus have the opposite ordering, $7/2^-$ being the lower one. Due to mixing with the $(3_1^- \otimes \pi 1 i_{13/2})$ configuration the "pure" $\pi 2f_{7/2}$ level is also pushed down, but only by about 0.2 MeV due to larger energy difference. Thus, after taking account of configuration mixing, not only the neutron $\Delta_{\ell s}^{(n)}(2f)$ splitting between the pure states increased as compared to 1.77 MeV, but also the proton $\Delta_{\ell s}^{(p)}(2f)$ splitting decreased to a smaller value.

Turning to the region of ^{132}Sn, we note that the corresponding experimental data on fragmentation of single-particle states are not known at present. However, as was pointed out by Blomqvist [20], the ^{132}Sn and ^{208}Pb nuclei are in some respect twins, having similar shell structures with the correspondence of $l \rightarrow l+1$, $j \rightarrow j+1$ for most of the orbitals in these regions. Therefore, all the arguments presented above for splitting of the $2f$ levels at ^{208}Pb are completely valid also for the $2d$ states at ^{132}Sn, with replacement of $1 i_{13/2}$ by $1 h_{11/2}$. So far there is no direct experimental data on the $B(E3; 3_1^- \rightarrow$ ground state) value in ^{132}Sn. However, the core has much higher rigidity here in comparison with ^{208}Pb and the energy of the 3_1^- state is substantially higher at 4.35 MeV. Thus from accounting for configuration mixing one expects some further increase of the $\Delta_{\ell s}^{(n)}(2d)$ splitting and a decrease of $\Delta_{\ell s}^{(p)}(2d)$, but these changes should be smaller than for the $2f$ levels at ^{208}Pb. Estimates based on an indirect evaluation of the $B(E3)$ value from the magnitude of the octupole effective charge in ^{134}Te [21] confirm the pattern of changes of the $\Delta_{\ell s}^{(p,n)}(2d)$ values presented above. However, in the absence of experimental data on direct reactions we present in tables 4 and 6 the values of energies at ^{132}Sn that do not include averaging over spectroscopic factors.

3 Theoretical approach

3.1 General considerations

Turning to the theoretical interpretation [4] of the experimental values of the spin-orbit splitting discussed above, we shall first recall that from the point of view of many-body theory the average spin-orbit potential has its origin in the pair spin-orbit interaction between nucleons (with tensor forces providing a minor contribution as well). On the level of qualitative arguments it was noted by Bohr and Mottelson [22] that due to the symmetry properties one should expect the neutron spin-orbit splitting to be somewhat larger than that for protons in heavier nuclei, simply due to a higher number of like particles in the neutron case. However, at that time the absence of experimental data did not permit a meaningful comparison

with measurements. With the presently available data we can fill this gap, providing also some quantitative considerations.

The two-body spin-orbit interaction differs from zero only in the states with a total spin $S = 1$. The neutron-neutron and proton-proton systems have the total isospin $T = 1$ and thus due to the Pauli principle have odd values of the relative orbital momentum L (in fact, $L = 1$). At the same time, the neutron-proton system is composed from the $T = 0$ and $T = 1$ states with equal weights, having $L = 0$ and $L = 1$, correspondingly. Due to the absence of spin-orbit interaction in states with $L = 0$, the pair spin-orbit np interaction is half as strong as that in pp or nn systems.

If $U_{\ell s}(n)$ and $U_{\ell s}(p)$ represent the magnitudes of the mean spin-orbit field for neutrons and protons and $\vartheta(T = 1, S = 1, L = 1)$ is a quantity representing the parameter of the pair spin-orbit interaction in a state with $T = 1, S = 1, L = 1$, then the above discourse leads to

$$U_{\ell s}(n) \sim \vartheta(1,1,1) \cdot \left(N + \frac{1}{2}Z \right) \equiv \vartheta \cdot \left(A - \frac{Z}{2} \right)$$

and

$$U_{\ell s}(p) \sim \vartheta(1,1,1) \cdot \left(\frac{N}{2} + Z \right) \equiv \vartheta \cdot \left(A - \frac{N}{2} \right). \quad (1)$$

As the spin-orbit splitting $\Delta_{\ell s}^{(n,p)} \sim U_{\ell s}(n,p)$, the relative *difference* "ε" of the neutron and proton spin-orbit splittings is given by the expression

$$\varepsilon = \frac{\Delta_{\ell s}^{(n)} - \Delta_{\ell s}^{(p)}}{(\Delta_{\ell s}^{(n)} + \Delta_{\ell s}^{(p)})/2} = \frac{2}{3} \frac{N - Z}{A}. \quad (2)$$

On the other hand, we can express the strength of the spin-orbit mean field in the form

$$U_{\ell s}(\tau_3) = V_{\ell s} \left(1 + \frac{1}{2} \beta_{\ell s} \frac{N - Z}{A} \cdot \tau_3 \right). \quad (3)$$

Here $\tau_3 = -1$ for neutrons, $\tau_3 = +1$ for protons and $\beta_{\ell s}$ is the parameter that defines the isospin dependence of the mean spin-orbit field. Then we easily obtain, this time in terms of eq. (3), an expression for the relative difference between the spin-orbit splittings of neutrons and protons in identical orbits, ε:

$$\varepsilon = -\beta_{\ell s} \frac{N - Z}{A}. \quad (4)$$

It follows from a comparison of eqs. (2) and (4) that $\beta_{\ell s} = -2/3$.

Strictly speaking, this derivation was performed for the two-body spin-orbit interaction. However, as mentioned above, tensor forces provide also some contribution to the spin-orbit splitting. This non-central interaction is proportional to S_{12} with

$$S_{12} = 3(\boldsymbol{\sigma_1 n})(\boldsymbol{\sigma_2 n}) - \boldsymbol{\sigma_1 \sigma_2} = \sqrt{24\pi} \cdot [[\sigma_1 \otimes \sigma_2]^2 \otimes Y_2]_0^0. \quad (5)$$

One can easily see from (5) that the diagonal matrix elements of this interaction are different from zero only for states with $S = 1$ and $L \geq 1$, of which the $S = T = L = 1$ one is of the main importance. It is just the state which was already considered in this subsection in the case of spin-orbit interaction. Consequently, the diagonal part of tensor forces also provides contribution of the type given by eq. (3) with $\beta_{\ell s} = -2/3$, and thus it leads only to a renormalization of the $V_{\ell s}$ value. However, as the spatial part of tensor operator is proportional to $Y_2(\mathbf{n})$ and due to the spin structure of S_{12}, this renormalization equals zero in cases of spin-saturated spherical nuclei. Thus in ^{16}O and ^{40}Ca tensor forces give a contribution to the isoscalar part of the spin-orbit splitting, that is mediated by their non-diagonal part and caused by admixtures, that are out of the Hartree-Fock–type ground state. As was shown in ref. [23], tensor forces may really lead to a substantial contribution to the isoscalar part of spin-orbit splitting. At the same time, in nuclei that are not spin saturated, such as ^{48}Ca, tensor forces can contribute to the spin-orbit splitting even in the "diagonal" scheme (*i.e.*, a scheme without admixtures), if the antisymmetrization is properly included. Our numerical calculations for seniority one states of ^{47}Ca and ^{47}K both having one neutron or proton hole and performed in the framework of the multiparticle shell model with tensor forces taken from our previous works [24–28], have demonstrated that the inclusion of a tensor component of the interaction leads to energy shifts that correspond to some variation of the spin-orbit splittings $\Delta_{\ell s}$, such that in ^{48}Ca $\Delta_{\ell s}^{(n)}(1d) - \Delta_{\ell s}^{(p)}(1d) = 0.34$ MeV and $\Delta_{\ell s}^{(n)}(1p) - \Delta_{\ell s}^{(p)}(1p) = 0.24$ MeV. These shifts arise from neutrons filling the $\nu 1 f_{7/2}$ subshell and are mainly due to charge exchange two-body matrix elements of the np-interaction mediated by the isovector part of the tensor force ($\sim \tau_1 \tau_2$). Thus the inclusion of tensor forces does not change the pattern of spin-orbit splitting, which also leads to negative values of $\beta_{\ell s}$ ranging from about -0.4 to -0.7. These results qualitatively agree with those presented in ref. [29], where in the framework of the Brueckner-Hartree-Fock method with Reid potential (containing both the spin-orbit and tensor components), a substantially larger neutron than proton splitting was obtained for the $1p$ and $1d$ orbitals in ^{48}Ca with $\beta_{\ell s}$ in the range from about -0.5 to -1.8. We note that the data on spin-orbit splittings of the $2d$ states in ^{132}Sn as well as on the splittings of the $2f$ and $3p$ levels in ^{208}Pb lead to effective values of $\beta_{\ell s}$ equal to -0.55, -0.60 and -0.27, respectively, which are numbers in very satisfactory general agreement with the prediction of eq. (4).

It is thus of substantial interest to evaluate to what extent the isotopic dependence of the spin-orbit splittings are reproduced by standard model calculations. Three different approaches were made as described below.

3.2 Evaluation I: Walecka model

The first evaluation is made in the Hartree approximation starting from the Dirac phenomenology with meson-

nucleon interactions according to the Walecka model [30]. One obtains (see for example [31–38] and references therein) a Skyrme-type single-particle equation for a nucleon having the effective mass m_N^*. This approach well explains the magnitude of spin-orbit splitting in nuclei. Here, and mainly for heavier nuclei, we concentrate only on the *difference* between the proton and neutron splittings of spin-orbit partners in the same nuclei and resulting from a spin-orbit potential having the form (see for example [34–37]):

$$\hat{U}_{\ell s} = \frac{\lambda_N^2}{2} \frac{1}{r} \left\{ \left(\frac{m_N}{m_N^*} \right)^2 \frac{\mathrm{d}}{\mathrm{d}r} [(V_\omega^0 - S_{\sigma,\sigma_0}^0) \right.$$
$$\left. -(V_\rho^1 - S_{\delta,\sigma,\sigma_0}^1) \cdot \tau_3] - 2k \left(\frac{m_N}{m_N^*} \right) \frac{\mathrm{d}}{\mathrm{d}r} V_\rho^1 \cdot \tau_3 \right\} \hat{\boldsymbol{\ell}} \cdot \hat{\mathbf{s}}, \quad (6)$$

where $V = V^0 - \tau_3 \cdot V^1$ and $S = S^0 - \tau_3 \cdot S^1$ are the vector and scalar fields related to corresponding mesons, $m_N^* = m_N + \frac{1}{2}(S - V)$, while k is the ratio of tensor to vector coupling constants of ρ-meson. Various approaches have been used to determine the coupling constants. In [37] the meson-nucleon coupling constants, defining the V and S fields, were taken from the Bonn NN boson exchange potential [39], where σ and σ_0 are scalar mesons imitating the 2π exchange in the NN systems with $T = 1$ and $T = 0$, correspondingly. In other works (see for example [34–36]) the constants were defined from the description of global nuclear properties, with inclusion of the σ^3 and σ^4 terms in the Lagrangian density (one σ-meson with the same characteristics for $T = 1$ and $T = 0$ channels was used, which leads to zero contribution of this meson to S^1 in formula (6); note also that the tensor term was not included in the ρ-meson vertex in refs. [34–36]).

Taking into account that the radial dependence of the (m_N/m_N^*) is much weaker than that of V and S, which are considered to be proportional to the density in the form of Fermi function, one can approximately present formula (6) as follows:

$$\frac{1}{r} \frac{\mathrm{d}f}{\mathrm{d}r} \cdot V_{\ell s} \left(1 + \frac{1}{2} \beta_{\ell s} \frac{N - Z}{A} \cdot \tau_3 \right) \hat{\boldsymbol{\ell}} \cdot \hat{\mathbf{s}};$$
$$f = \left[1 + \exp \left(\frac{r - R}{a} \right) \right]^{-1}. \quad (7)$$

Calculating the V and S magnitudes in the center of nuclei at the values of vector and scalar densities $\rho_v = 0.17$, $\rho_s = 0.16$, $\rho_v^- = 0.17\,(N - Z)/A$, $\rho_s^- = 0.16\,(N - Z)/A$ (all in fm^{-3}), using the coupling parameters from [37,39] and taking into account the isotopic dependence of m_N/m_N^*, we obtain $V_{\ell s} \approx 34$ MeV· fm^2 and $\beta_{\ell s} \approx -0.40$. If we use the NL2 set of parameters from [35,36] then we have $V_{\ell s} \approx 31$ MeV · fm^2, $\beta_{\ell s} \approx -0.43$. At the same time the set NL1 from [34,36], giving small values of effective masses, leads to $V_{\ell s} \sim 50$ MeV · fm^2 and $\beta_{\ell s} \sim -1.3$. As the V^1, S^1 magnitudes are proportional to ρ_v^- and ρ_s^-, both the formulae (6) and (7) give equal spin-orbit splitting for protons and neutrons in the $N = Z$ nuclei. It should be noted, that the value of $\beta_{\ell s}$ is always

negative and is determined mainly, or entirely, by the ρ-meson contribution.

The magnitudes of the empirical effective values of $\beta_{\ell s}$ at ^{132}Sn and ^{208}Pb, listed in subsect. 3.1, are quite well reproduced by the model calculations in this subsection, in particular by those using the first two sets of parameters.

It is worth mentioning that a study of the neutron spin-orbit splitting in light nuclei as a function of A at given Z was recently performed in the framework of the Walecka model by Lalazissis *et al.* [40]. However, the intercomparison between the splittings of both proton and neutron "similar" spin-orbit doublets in the same nuclei was not performed there.

3.3 Evaluation II: Woods-Saxon model

In the second approach, using a Woods-Saxon (W-S) model, we let the single-particle levels be generated by the potential

$$\hat{U}(r, \hat{\sigma}, \tau_3) = U_0(\tau_3) f(r)$$
$$+ \frac{U_{\ell s}(\tau_3)}{r} \frac{\mathrm{d}f}{\mathrm{d}r} \hat{\boldsymbol{\ell}} \cdot \hat{\mathbf{s}} + \frac{(1 + \tau_3)}{2} U_{\mathrm{Coul}}, \quad (8)$$

where $U_0(\tau_3) = V_0(1 + \frac{1}{2}\beta \frac{N-Z}{A} \cdot \tau_3)$; $U_{\ell s}$ and $f(r, a, R)$ are defined by eqs. (3) and (7), $R = r_0 A^{1/3}$, while $U_{\mathrm{Coul}}(r, R_c, Z)$ represents the potential of a uniformly charged sphere with the charge Z and radius $R_c = r_c A^{1/3}$.

In previous works [24–28], calculations were made using the $V_0 = -51.5$ MeV, $r_0 = 1.27$ fm, $V_{\ell s} = 33.2$ MeV·fm^2, $a(p) = 0.67$ fm, $a(n) = 0.55$ fm and $\beta_{\ell s} = \beta = 1.39$, which on the average described the spectra of single-particle states in nuclei from ^{16}O to ^{208}Pb. This set of parameters is denoted here as the "Stnd" (standard) one. With the appearance of new experimental data on the single-particle levels, we performed a new determination of parameter values through the Nelder-Mead method [41] by minimizing the root-mean square deviation

$$\delta = \sqrt{\frac{1}{n} \sum_{k=1}^{n} (\varepsilon_k^{\mathrm{theor}} - \varepsilon_k^{\mathrm{exp}})^2}. \quad (9)$$

The computation demonstrated a very small sensitivity of results to the value of r_c, which was adopted to be the same as before: $r_c = 1.25$ fm. The minimization of δ performed for all nuclei presented in tables 1–5 with $r_c = 1.25$ fm and different values of r_0, showed that the minimum in all cases corresponds to $r_0 \approx 1.27$ fm, that also coincides with the value adopted by us before. The values $r_c = 1.25$ fm and $r_0 = 1.27$ fm were thus fixed in further calculations.

As was noted above, the optimal relation of proton to neutron spin-orbit splitting corresponds to $\beta_{\ell s} \sim -0.6$. The fourth column, "Set 1", of tables 4 and 5 presents the values of theoretical energy levels obtained in the optimization with fixed values of $\beta_{\ell s} = -0.6$, $a_p = 0.67$ fm and $a_n = 0.55$ fm.

The fifth column, "Set 2", of tables 4 and 5 presents the results of optimization with only two fixed parameters: $a_p = 0.67$ fm and $a_n = 0.55$ fm.

The values of "Set 3" corresponds to an optimization at fixed $\beta_{\ell s} = -0.6$, while "Set 4" are the results with no parameters fixed.

We see that the optimized values of V_0, $V_{\ell s}$ and β (see formula (8)) are very close to the "Stnd" ones, with small variations from nucleus to nucleus. The magnitudes of the diffusinesses "a" vary more strongly, differing by about 10% to 15% from their "Stnd" values. A comparison of the "Stnd" with "Set 1" and of "Set 3" with "Set 4" results shows that the contribution of $\beta_{\ell s}$ to the root-mean square deviation δ is small. It is thus more reasonable to define $\beta_{\ell s}$ not from a minimization of δ, but rather by using the experimental and theoretical arguments mentioned above. This conclusion is confirmed by the results of Koura and Yamada [42], who made a number of different fits of W-S parameters to the same set of experimental data, obtaining diverse (in magnitude and sign) values of the parameter that defines the contribution to the spin-orbit term, which is linear in $(N - Z)/A$. A global adjustment of W-S parameters simply appears to be only weakly sensitive to details of the spin-orbit splitting.

As mentioned previously, the energies of levels in nuclei with $N = Z$ (see tables 1–3) are independent of β and $\beta_{\ell s}$. Here the optimization was performed twice, first with fixed values of $a_n = 0.55$ fm and $a_p = 0.67$ fm with a subsequent definition of V and $V_{\ell s}$ ("Set 1") and secondly without fixing any parameters ("Set 3").

The results of the calculations presented in tables 1 to 5 include some levels having positive energies, *i.e.* unbound but sub-barrier states. In such cases we present here the real part of the single-particle energies only for those states having very small decay widths.

To summarize evaluation II, we have determined the parameters of the W-S potential using a global mean square-root optimization, except for the isospin-dependent spin-orbit term, where the parameter value was found to be insensitive to the adjustment. Hence the value of $\beta_{\ell s} \sim -0.6$ was deduced from physical considerations based on experimental spin-orbit splittings.

3.4 Evaluation III: Hartree-Fock with a Skyrme interaction

For the third-model approach, which complements the first two evaluations using the empirically adjusted W-S potential (8) and the microscopical procedure, we have selected the Hartree-Fock calculations with the SIII interaction. The results of these self-consistent calculations, listed in the last two columns of tables 1 to 5, were obtained by considering the contribution of a single-particle part of the center-of-mass energy and taking into account the Coulomb exchange term in the Slater approximation. The SIII-1 results correspond to calculations which take into account all terms of the energy functional contributing to spin-orbit splitting, while the SIII-2 results have been obtained by omitting the spin density terms in

the spin-orbit potential. In the last case our results are close to those from the study by Leander *et al.* [43] performed for ^{208}Pb, ^{132}Sn and ^{100}Sn nuclei. We see that the results obtained in the framework of the Hartree-Fock method also demonstrate that the calculated neutron spin-orbit splittings of the $2d$ orbit in ^{132}Sn as well as of the $2f$ and $3p$ orbits in ^{208}Pb are larger than for protons and they correspond to effective $\beta_{\ell s}$ in the interval of -0.9 to -0.6. We note that the difference between the neutron and proton spin-orbit splittings is reproduced here by using a simple parameterization of Skyrme forces. Our calculated results differ from those of Noble [32] who proposed that the isotopic dependence of the spin-orbit potential in the Hartree scheme is cancelled through the contribution of exchange terms, but agree with that of [29]. We mention here that the SIII parameterization contains density-dependent terms that imitate in some sense the three-body interaction.

Tables 1 and 2 give results for ^{16}O and ^{40}Ca, which are spin-saturated nuclei. In these cases the spin density terms, included in SIII-1 but not in SIII-2, do not contribute significantly to the spin-orbit splitting (the contributions in these cases are only due to small differences in the radial wave functions of spin-orbit partners). Consequently, as can be expected, the SIII-1 and SIII-2 calculations give very similar results in both cases.

4 Discussion and conclusions

Using theoretical analysis and systematics of available experimental data we have derived formula (3) that describes the *difference* between the neutron and proton spin-orbit splittings, *i.e.* the isotopic dependence of the mean-field spin-orbit splitting. The splitting becomes larger for neutrons than for protons in nuclei having $N > Z$. The general arguments presented initially (based on the properties of the two-body spin-orbit and tensor interactions) gave a result in fair agreement with the empirical observations. A further microscopic study within the Walecka model supports this initial result, while it was found that a global fit of Woods-Saxon model parameters appears to be rather insensitive to the isotopic dependence of the spin-orbit splitting. A self-consistent calculation using the SIII interaction gave results in general agreement with the experiment and prediction by eq. (3) with negative values of $\beta_{\ell s} \sim -0.6$.

In this context, one should point out that within the Walecka model, the sign of the isospin term in the spin-orbit potential is in agreement with the sign of an analogous term present in the expression for the central nuclear potential. While the spin-orbit term in this model is defined, very approximately, by the $(V - S)$ combination of the entering fields, the central nuclear potential is proportional to the $(V + S)$ combination. The main, isoscalar, part of the $(V - S)$ term is positive and the addition of an isovector contribution, arising from V^1, leads for the $N > Z$ nuclei, as was shown above by us, to an additional term (positive for neutrons and negative for protons), its

magnitude growing with $(N - Z)$, together with the ratio of neutron to proton splittings. At the same time, the isoscalar part of the central $(V + S)$ term is negative. The addition of a V^1 term leads here for neutrons in $(N > Z)$ nuclei to reduction of the absolute value of $(V + S)$. So, with increasing N at a given Z, the depth of the central nuclear potential for neutrons decreases and they become less bound, while the protons become more bound. All this is reflected in the W-S model (see eq. (8) above) by the fact that $\beta_{\ell s}$ is negative, while β is positive. The two models are thus fully consistent in this respect.

The isotopic dependence of the spin-orbit splitting has also been studied with methods somewhat different than those used here. In the work of Mairle [44] the average spin-orbit potential was obtained as a convolution with proton and neutron densities taken in the ratio defined by the short-range two-body spin-orbit interaction. However, the isotopic dependence of the average spin-orbit potential was not derived here in an explicit form. This point has some importance, since our analysis, based on the existing empirical data and different theoretical approaches, resulting in a simple expression, immediately shows that the difference between the neutron and proton splittings becomes saturated at large N, which precludes very large differences. The rather modest difference with a magnitude of about 10% seen in the ^{132}Sn region is already about 25% of the saturation value, suggesting that the isospin dependence in itself is unlikely to lead to dramatic structural changes. However, in cases of extreme neutron excess, when the difference between neutron and proton spin-orbit splittings approaches the maximum value of about 40% (corresponding to several hundreds of keV) a rather significant effect on the ordering of levels can be expected.

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